

# A Mathematical Structure for Modeling Inventions

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## Abstract

The paper is the first of several ones [63,73] describing a mathematical structure developed in the FSTP project, mathematically modeling Substantive Patent Law ("SPL") and its US Highest Courts' precedents primarily for emerging technologies inventions. Chapter 2 presents this mathematical structure comprising particularly 3 abstraction levels - each comprising "inventive concepts", their "subset coverings", "concept transformations", "induced concept relations", and "refinements". Chapters 3 and 4 explain its practical application in describing an invention respectively testing it by an Innovation Expert System (IES) for its satisfying SPL.

Using the notion of "inventive concepts" for precisely describing emerging technologies inventions has been introduced into SPL precedents by the US Supreme Court during its ongoing "SPL initiative" - marked by its KSR/Bilski/Mayo/Myriad decisions. It induced, into the FSTP project, a rigorous mathematical analysis of allegedly new problems caused by these Highest Courts' SPL decisions about emerging technologies inventions. This analysis proved extremely fertile by enabling not only clarifying/removing obscurities in such problems but also developing powerful "patent technology" in the FSTP project.

## 1 Introduction

This FSTP project paper addresses the community of mathematicians not that much interested in the most recent problems in SPL precedents about emerging technologies inventions, but in contributing to its scientification in an unquestionable manner, i.e. in exerting rigorous mathematical scrutiny to it. It hence deals with providing a mathematical fundament - established by a sophisticated mathematical structure - for a very topical area of greatest socio/economic importance for the world's wealthy countries, as controlling the flow of annually several 100 billions of US Dollars [67], just as for supporting the transfer into emerging countries the know-how about innovativity enabling this wealth.

The FSTP project comprises, on the one side, a range of publications about developing the “patent/innovation technology” for a cutting edge prototype, the “Innovation Expert System (IES)”, as technologically today possible. In the future such IESes will be indispensable for the efficiency of the everyday professional activities of the hundred thousands of patent/innovation professionals of all kinds - in particular researchers/inventors, their R&D managers, R&D investors, patent/license lawyers, patent examiners, judges, product managers, marketing managers, . . . , in all kinds of emerging technologies, be it telecommunications/nano/genetics/drugs/. . . . /business/sports/. . . technologies. These publications deal with leveraging on the capability of precisely describing emerging technologies inventions, as required by the US Supreme Court’s SPL precedents during its ongoing “SPL initiative”, i.e. its famous line of KSR/Bilski/Mayo/Myriad decisions. It thus induced, on the FSTP project, a rigorous mathematical analysis of the thereby arising new decision problems in SPL precedents. Its scrutiny proved extremely fertile and enabled clarifying and removing the notional and legal obscurities in SPL stirred up by emerging technologies, i.e. having been lingering in SPL since ever, as well as developing “patent/innovation technology” based on that scientific approach to creativity/innovativity.

On the other side, there are FSTP publications just as this one, elaborating on the mathematical foundation of this new technology for ascertaining its well-definedness, e.g. for excluding allegedly correct legal statements about an invention but evidently contradicting each other - as it recently repeatedly occurred in the CAFC. The research results published by them deal with groundbreaking mathematical/theoretical modeling issues by Advanced IT for enabling these practical developments in the area of stimulating/supporting/protecting/commercializing such emerging technology innovations.

This paper, in particular, reports about a sophisticated mathematical structure enabling deciding in an unquestionable as mathematically assured - and hence consistent and predictable - way about the patent eligibility and patentability of a developing/claimed/reexamined/licensed/infringed/. . . invention represented by a patent/application/contract/. . . . To this end, this new mathematical structure has been designed such as to facilitate precisely modeling the interpretation of SPL by the Highest Courts and their respective precedents as to emerging technologies inventions, just as the technical/factual needs arising from such inventions. The established classical interpretations of SPL in all industrial nations are strongly depending on the tangibility/visibility of inventions and hence prove vastly deficient when dealing with emerging technologies inventions, as these are virtually always intangible and invisible, i.e. “model based”. They hence may be “indefinite” and/or “preemptive” as their scopes of patent monopoly are ambiguous resp. potentially comprise inventions not yet known at all when granting them - or just socially unwanted, e.g. as felt threatening - all these being new reasons of their non-patent-eligibility.

But the main purpose the here presented mathematical structure must serve, for emerging technologies inventions just as for classical ones, is to facilitate modeling the metes and bounds of any patent analysis and achieving its objectives, namely to support deciding correctly and semi-automatically whether it satisfies SPL. In its above mentioned SPL initiative, more precisely in its Mayo decision, the US Supreme Court introduced into SPL precedents the key notion of “inventive concepts” and required using them for construing, for any claimed emerging technologies invention its (thus refined) claim construction, being a shorter way than saying “using its inventive concepts in checking whether it meets the requirements stated by SPL” - as explained in legal terms in [18,19] and will mathematically be presented in detail in this and the following mathematical paper(s). [73]

The definition of the key notion “inventive concept” as discussed in the FSTP Project - and mathematically described precisely in Chapter 2 of this paper and leading to its mathematical structure - is a dramatic simplification of the much more powerful notion of “concept” used in AI since about 50 years, today being the basis of Advanced IT branches such as “Description Logic” and “Mathematical Knowledge Representations” [2-4]. Here, the notion of “inventive concept” is customized for modeling just SPL precedents, nothing else. While in DL or KR concepts serve for modeling recursively building compound concepts out of simpler ones, the Highest Courts’ SPL precedents proceed, by contrast to Advanced IT, the opposite way: For mathematically modeling the properties of the elements of a claimed invention precisely, their compound inventive concepts (modeling their compound properties, as usually used for describing/claiming inventions) must be disaggregated into conjunctions of elementary inventive concepts (modeling their elementary properties) - as precisely reasoning about its satisfying SPL by using compound inventive concepts is absolutely impossible (though many patent practitioners refuse to recognize this - in spite of all scientific insights of Advanced IT into this problem). This indispensable disaggregation needs the clear rigorous mathematical structure presented by Chapter 2, as explained in Chapter 3 - for enabling conducting the invention’s unquestionable test for its satisfying SPL, as outlined in Chapter 4. I.e.: For an emerging technologies invention only this clear mathematical structure enables to conduct its test under SPL such that hitherto unavoidable and irresolvable obscurities therein may be clarified in a rational way.

To conclude and summarize these introductory remarks: The aim of this paper is to present in mathematical rigor the mathematical structure developed in the FSTP project as fundament of any IES, not just the present IES prototype, and to briefly indicate how this mathematical frame work is motivated by and related to the everyday legal business of patent professionals. All publications about the FSTP project and its IES prototype are available on [www.fstp-expert-system.com](http://www.fstp-expert-system.com).

## 2 Concepts and Mathematical Structure

The US Supreme Court's Mayo decision requires that claimed inventions are described by inventive concepts when testing them under SPL, in particular if these claimed inventions are from emerging technologies areas. The FSTP project developed the mathematical framework for using inventive concepts for testing a claimed invention, described by its inventive concepts, under SPL. While the headlines of the FSTP publications on [www.fstp-expert-system.com](http://www.fstp-expert-system.com) explain, which of them deal with the SPL test of a claimed invention, this Chapter will develop step by step the formal definition of inventive concepts - a set of which describes this invention precisely and completely - and the mathematical structures imposed on the set of all of them by testing it under SPL and then representing this test. I.e., this total mathematical structure models the transformation of the invention's non-operational SPL test into the operational and legally/factually equivalent FSTP-Test (see FIG 1).

In principle, an inventive concept, in-C, is a pair of a legal concept and a technical alias factual concept, i.e.  $\text{in-C} = (\text{le-C}, \text{cr-C})$ , the latter one supposed to be creative, hence its name. In this Chapter totally focused on mathematical structures, only their creative concepts are considered, and for further simplification they are temporarily called just concepts - in paragraph 9 these simplifications are elaborated on. In any practical application this bisection of inventive concepts must be preserved, for the sake of the quite different beings of the notions of creative and legal concepts, i.e. for thereby establishing a legal and factual clarity often disregarded hitherto.

1. As a first step we define the mathematical structure for a description of a concept:

**Definition 1:** A concept  $C$  is given by a triple of data

$$C = (DC, VC, UC),$$

where  $DC$  is a non-empty set called the *domain* of  $C$ ,  $VC$  is a non-empty finite set of non-empty sets  $vC \in VC$ ,  $VC$  being called the *set of value sets*  $vC$  of  $C$  and where a  $vC$  is called a *value set* of  $C$ ,  $UC$  is a map from  $DC$  to  $VC$ , represented by a non-empty relation from  $DC$  to  $VC$ , i.e.,  $UC \subseteq DC \times VC$ , each  $d \in DC$  is related to some  $vC \in VC$  and no  $d \in DC$  is related to two different value sets in  $VC$ .  $UC$  is called the *universe* of  $C$ .

A concept is called *binary concept* if  $VC$  has exactly two value sets, one being identified with  $T = \text{True}$  and the other with  $F = \text{False}$ . This is abbreviated by  $VC = \{T, F\}$ . In the following only **binary concepts** will be considered.

2. A binary concept has three equivalent descriptions:

i) The usual one as a triple  $C = (DC, \{T, F\}, UC)$ , where  $DC$  is the domain of the concept and  $UC$  is a relation between  $DC$  and  $\{T, F\}$ . As defined above  $UC$  has to satisfy the following requirements: a) each  $d \in DC$  is related to some

$vC \in \{T, F\}$  and b) no  $d \in DC$  is related to  $T$  and  $F$ .

ii) A binary concept may be expressed by a triple  $C = (DC, MC, \{T, F\})$ , where  $MC : DC \rightarrow \{T, F\}$  is the map connected with  $UC$  by  $UC = \{(d, MC(d)) \mid d \in DC\}$ .

iii) The third description is given by the separation of the domain  $DC$  into the *truth-set*  $TS(C)$ , being the complete preimage of  $T$  under  $MC$ , and the *false-set*  $FS(C)$  being the complete preimage of  $F$  under  $MC$ , i.e.  $C = (DC, TS(C), FS(C))$ , where  $DC = TS(C) \cup FS(C)$  and  $TS(C) \cap FS(C) = \emptyset$ .

These descriptions are linked by the following equations:

$$UC = (TS(C) \times \{T\}) \cup (FS(C) \times \{F\}),$$

where the universe  $UC$  now also is represented as a set, and the sets  $TS(C) \times \{T\}$  and  $FS(C) \times \{F\}$  represent a partition of  $UC$  into two disjoint subsets,  $MC$  decomposes into two constant maps  $MC|_{TS(C)}$  having the value  $T$  and  $MC|_{FS(C)}$  having the value  $F$ , and  $DC = p_1(UC)$ ,  $p_1$  denoting the projection of  $DC \times \{T, F\}$  onto its first factor  $DC$ .

In the following we will change from one description to another depending on what will be suitable for the current considerations.

3. Today, everyday business of a patent practitioner deals only with information in natural language or simple graphics representation, which the posc considers to be lawfully disclosed by the document providing it (whereby the posc represents the fictional “person of ordinary skill and creativity” pertinent alias familiar to the subject area of the patent preparation/application/prosecution/licensing - at issue, i.e. is the perfect representative of the pertinent ordinary skill and creativity, and hence of no extraordinary qualification). Advanced IT (e.g. Semantics, Natural Language, Knowledge Representation, Compiler/Interpreter research) tells that on this information representation “level” precise statements are impossible. For deriving from this imprecise original information representation a precise one, two preciseness increasing steps are indispensable in FSTP alias Patent Technology today to be performed by the posc:

- Firstly, transforming the original and NL syntax based information representation, e.g. of a patent specification, into NL terms but some FOL syntax based information, and
- secondly transforming the latter representation using compound inventive concepts as “alphabet” (consisting of these NL terms) - by preserving its exact FOL syntax - into a refined knowledge representation, again using NL terms, but refined ones such as to achieve that any of these non-refined NL terms is logically equivalent to the conjunction of certain ones of these refined NL terms (in total defining the refined alphabet).

In order to link to each other these three different kinds/representations of inventive concepts of (often) different preciseness of an invention - as generated by the posc by its refinement process - we need transformations between the sets of such associated binary concepts. These transformations are given by

bijections between underlying subset coverings of the union of the universes for each concept set. Subset coverings are defined by the following:

**Definition 2:** Let  $Cset$  be a finite set of concepts. A finite *subset collection* of the union of the universes of the concepts belonging to  $Cset$  is given by a finite sequence  $SSU = (ssU_1, \dots, ssU_\Lambda)$  of  $\Lambda$  non-empty sets

$$ssU_\lambda \subseteq \bigcup \{UC \mid C \in Cset\}$$

whereby the union is considered as (concept-wise) disjoint. Such a subset collection is called a finite *subset covering*  $SSCov = (ssCov_1, \dots, ssCov_\Lambda)$  of the union of the universes of the concepts in  $Cset$ , iff the following covering condition is satisfied:

$$\bigcup \{ssCov_\lambda \mid 1 \leq \lambda \leq \Lambda\} = \bigcup \{UC \mid C \in Cset\}.$$

The choice of a finite sequence of subsets for the collection instead of just choosing a finite number of subsets is motivated by the fact that for the transformation to be defined below some subsets may have to be considered several times, which requires to distinguish several copies of the same set formally for these cases. We may describe a subset covering in the form  $SSCov = \{ssCov_1, \dots, ssCov_\Lambda\}$ , if multiple copies of the same set do not occur.

4. Every element  $ssCov_\lambda$  of a subset covering  $SSCov = (ssCov_1, \dots, ssCov_\Lambda)$  of the union of the universes of the concepts in  $Cset$  can be interpreted as the universe of a binary concept  $CssCov_\lambda := (DCssCov_\lambda, \{T, F\}, ssCov_\lambda)$ , where  $DCssCov_\lambda$  still has to be defined. The following equations and inclusions show that  $ssCov_\lambda$  is a relation between  $\bigcup \{DC \mid C \in Cset\}$  and  $\{T, F\}$ :

$$ssCov_\lambda = ssCov_\lambda \cap (\bigcup \{UC \mid C \in Cset\}) = \bigcup \{ssCov_\lambda \cap UC \mid C \in Cset\} \subseteq \bigcup \{DC \times \{T, F\} \mid C \in Cset\} = (\bigcup \{DC \mid C \in Cset\}) \times \{T, F\}.$$

For the domain, which has not been defined so far, we get

$$DCssCov_\lambda := \bigcup \{p_1(ssCov_\lambda \cap UC) \mid C \in Cset\},$$

where  $p_1$  denotes the projection onto the first factor of a product of sets. The partition of  $DCssCov_\lambda$  into a truth- and a false-set is given by

$$TS(CssCov_\lambda) := \bigcup \{p_1(ssCov_\lambda \cap UC) \cap TS(C) \mid C \in Cset\} \text{ and } FS(CssCov_\lambda) := \bigcup \{p_1(ssCov_\lambda \cap UC) \cap FS(C) \mid C \in Cset\}.$$

**5. Definition 3:** Let  $Cset$  and  $Cset'$  be two sets of concepts equipped with subset coverings  $SSCov$  and  $SSCov'$  of their unions of universes. A *concept transformation* between  $Cset$  and  $Cset'$  is given by a bijection  $Z$  from  $SSCov$  to  $SSCov'$ . Hence we have the consequences:

- i) The lengths  $\Lambda$  and  $\Lambda'$  of  $SSCov$  and  $SSCov'$  are the same.
- ii) There is a permutation  $\sigma$  of the natural numbers from 1 to  $\Lambda$  such that  $Z(ssCov_\lambda) = ssCov'_{\sigma(\lambda)}$  for all  $1 \leq \lambda \leq \Lambda$ .

**Remarks:** i) After a suitable relabeling of  $SSCov$  or  $SSCov'$  we may assume that  $Z(ssCov_\lambda) = ssCov'_\lambda$  for all  $1 \leq \lambda \leq \Lambda$ . But that requires that the numberings of  $SSCov$  and  $SSCov'$  are not subject to other conditions.

- ii) Each concept transformation has an inverse given by  $Z^{-1}(ssCov'_\lambda) = ssCov_{\sigma^{-1}(\lambda)}$ .

6. **Definition 4:** A concept transformation  $Z$  between a concept set  $Cset$  and a concept set  $Cset'$  leads to an *induced concept relation*  $Ind(Z)$  from  $Cset$  to  $Cset'$  as follows:  $(C, C') \in Ind(Z) \subseteq Cset \times Cset'$ , if and only if there exists a  $\lambda$ ,  $1 \leq \lambda \leq \Lambda$ , such that  $UC \cap ssCov_\lambda \neq \emptyset$  and  $UC' \cap Z(ssCov_\lambda) = UC' \cap ssCov'_{\sigma(\lambda)} \neq \emptyset$ .

**Remarks:** i) The induced concept relation for  $Z^{-1}$  is given by the inverse relation of  $Ind(Z)$ ,  $Ind(Z^{-1}) = (Ind(Z))^T$ .

ii) For the next paragraph we have to investigate the case that the **induced relation**  $Ind(Z)$  on the concept level **is a map**. This implies extra properties for  $Z$ . The uniqueness of the values of the map  $Ind(Z)$  leads to the condition, that for each  $C \in Cset$  and each  $\lambda$ ,  $1 \leq \lambda \leq \Lambda$ , satisfying  $UC \cap ssCov_\lambda \neq \emptyset$ , we have  $UC' \cap Z(ssCov_\lambda) = UC' \cap ssCov'_{\sigma(\lambda)} = \emptyset$  for all  $C' \in Cset' \setminus \{(Ind(Z))(C)\}$ .

This implies that  $ssCov'_{\sigma(\lambda)}$  is a subset of the universe of  $(Ind(Z))(C)$ . Hence  $SSCov'$  consists of subsets of the universes of the concepts in  $Cset'$  and splits up into mutually disjoint subsequences, each of them consisting of subsets of the universe of the same concept in  $Cset'$ . The covering condition for  $SSCov'$  implies, that the subsequence of those  $ssCov'_\tau$ , which are contained in the universe  $UC'$  of a fixed  $C' \in Cset'$ , are covering  $UC'$ .

Another consequence is that  $UC_0 \cap ssCov_\lambda \neq \emptyset$  implies  $UC \cap ssCov_\lambda = \emptyset$  for all  $\{C \in Cset \mid (Ind(Z))(C) \neq (Ind(Z))(C_0)\}$ , i.e.,  $SSCov$  splits up into mutually disjoint subsequences, such that the sets in each subsequence are subsets of the union of the universes of those concepts in  $Cset$ , which have the same image under  $Ind(Z)$ . Hence the concept transformation given by  $Z^{-1}$  may be called a *refining transformation* and  $Cset$  may be called a *refinement* of  $Cset'$ .

7. **Definition 5:** Let  $Cset$  and  $Cset'$  be two sets of concepts equipped with subset coverings  $SSCov$  and  $SSCov'$  of their unions of universes,  $Z$  a concept transformation between  $Cset$  and  $Cset'$ , given by a bijection  $Z$  from  $SSCov$  to  $SSCov'$ , let

$$G_\lambda : DCssCov_\lambda \longrightarrow DCssCov'_{\sigma(\lambda)}, \quad 1 \leq \lambda \leq \Lambda,$$

be a system of maps between the domains of the covering sets, where  $Z(ssCov_\lambda) = ssCov'_{\sigma(\lambda)}$  and where the sets of the coverings are interpreted as concepts (see paragraph 4). Then the transformation given by  $Z$  and the system of maps  $G_\lambda$ ,  $1 \leq \lambda \leq \Lambda$ , are called *truth-preserving*, if

$$G_\lambda(TS(CssCov_\lambda)) = TS(CssCov'_{\sigma(\lambda)}), \quad 1 \leq \lambda \leq \Lambda, \text{ and}$$

$$G_\lambda(FS(CssCov_\lambda)) = FS(CssCov'_{\sigma(\lambda)}), \quad 1 \leq \lambda \leq \Lambda.$$

These conditions imply that  $G_\lambda$  is surjective for all  $1 \leq \lambda \leq \Lambda$ .

**Remark:** A weaker version of the property to be truth-preserving is given by the conditions that

$$G_\lambda(TS(CssCov_\lambda)) \subseteq TS(CssCov'_{\sigma(\lambda)}), \quad 1 \leq \lambda \leq \Lambda, \text{ and}$$

$$G_\lambda(FS(CssCov_\lambda)) \subseteq FS(CssCov'_{\sigma(\lambda)}), \quad 1 \leq \lambda \leq \Lambda.$$

Also in the general case that the  $G_\lambda$  are relations, which are not maps, a similar definition of truth-preserving can be given.

8. In connection with a concept transformation  $Z$  between a concept set  $Cset$  and a concept set  $Cset'$  we have the possibility to enhance the concepts in  $Cset$  by parts of concepts in  $Cset'$ . Such an extension is constructed as follows:

i) As a first step the induced relation  $Ind(Z)$  of a concept transformation is used to enhance each concept  $C = (DC, \{T, F\}, UC)$  in  $Cset$  to an *extended concept*  $C_e = (DC_e, \{T, F\}, UC_e)$  in the following way:

$$DC_e = DC \dot{\cup} (\dot{\bigcup} \{p_1(UC' \cap ssCov'_{\sigma(\lambda)}) \mid 1 \leq \lambda \leq \Lambda \text{ and } (C, C') \in Ind(Z)\}),$$

$$UC_e = UC \dot{\cup} (\dot{\bigcup} \{UC' \cap ssCov'_{\sigma(\lambda)} \mid 1 \leq \lambda \leq \Lambda \text{ and } (C, C') \in Ind(Z)\}).$$

ii) Having extended all concepts in  $Cset$  in this way, we get the *extension*  $Cset_e$  of  $Cset$ . Simultaneously we get the extended subset covering  $SSCov_e$  for  $Cset_e$  defined by  $(ssCov_e)_\lambda = ssCov_\lambda \dot{\cup} ssCov'_{\sigma(\lambda)}$  for all  $1 \leq \lambda \leq \Lambda$ . The extended concept transformation  $Z_e$  is given by just selecting the second set from the disjoint union, i.e.  $Z_e((ssCov_e)_\lambda) = ssCov'_{\sigma(\lambda)} = Z((ssCov)_\lambda)$ .

Everything just mentioned also holds, if the above restriction "... by parts of concepts in  $Cset''$ " is left away. Then it becomes evident that the simplification performed at the beginning of paragraph 1, of considering of inventive concepts just their therein embedded creative concepts, is easily reversed: Simply by extending any inventive concept - wherever it occurs - in the way just described by its legal concept component. An alternative way of looking at that phenomenon is to ignore the just quoted initial simplification as well as that an inventive concept is a pair of a creative concept and a legal concept - as further going explained in Chapter 3.

To put it in other words: Any of the 3 components DC, VC, UC of an inventive concept is a pair of a technical/factual space and a legal space. For facilitating grasping the elaborations of this Chapter, we initially abstracted and may abstract also in what follows from the inventive concepts being bifid - i.e. mentally just leave their bisection aside - though in the end both spaces are indispensable for matching the needs of patent jurisprudence, in particular Highest Courts' SPL precedents.

9. Important for these knowledge representations and their transformations is also, which restriction of concept sets and concept transformations are induced by the needs arising from the mathematical structures representing a patent, more precisely: the legally and technically in-depth description of an invention/innovation - also as explained in Chapter 3. Let  $Z$  be a concept transformation between the concept set  $Cset$  and the concept set  $Cset'$  given as a bijection from a subset covering  $SSCov$  for  $Cset$  to a subset covering  $SSCov'$  for  $Cset'$ . Let  $Cset'_r$  be a subset of  $Cset'$ . We assume without loss of generality that

$$(ssCov')_\lambda \cap \dot{\bigcup} \{UC' \mid C' \in Cset'_r\} \neq \emptyset \text{ for all } 1 \leq \lambda \leq \Lambda_r \text{ and}$$

$$(ssCov')_\lambda \cap \dot{\bigcup} \{UC' \mid C' \in Cset'_r\} = \emptyset \text{ for all } 1 + \Lambda_r \leq \lambda \leq \Lambda.$$

Then the sets  $(ssCov'_r)_\lambda \cap \dot{\bigcup} \{UC' \mid C' \in Cset'_r\}$ ,  $1 \leq \lambda \leq \Lambda_r$  provide a subset covering  $SSCov'_r = \{(ssCov'_r)_\lambda \mid 1 \leq \lambda \leq \Lambda_r\}$  for  $Cset'_r$ , called the *restriction* of  $SSCov'$  to  $Cset'_r$ . Applying the inverse induced concept relation  $(Ind(Z))^T$  to  $Cset'_r$ , i.e. looking for all  $C \in Cset$  being in relation  $Ind(Z)$  to concepts in  $Cset'_r$ ,



we get the *preliminary restriction*  $Cset_{pr} := (Ind(Z))^T(Cset'_r)$  of  $Cset$  corresponding to  $Cset'_r$ . Finally we get with  $\sigma$  as in Definition 4 the restriction  $SSPCov_r$  of  $SSCov$  by setting  $(ssPCov_r)_\lambda := ssCov_{\sigma^{-1}(\lambda)} \cap \bigcup \{UC \mid C \in Cset_r\}$  for all  $1 \leq \lambda \leq \Lambda_r$ . The definition of  $Ind(Z)$  implies that  $(ssPCov_r)_\lambda \neq \emptyset$  for all  $1 \leq \lambda \leq \Lambda_r$  and even more: For every  $C \in Cset_r$  there exists a  $\lambda$ ,  $1 \leq \lambda \leq \Lambda_r$  such that  $(ssPCov_r)_\lambda \cap UC \neq \emptyset$ . Setting  $Z_r((ssPCov_r)_\lambda) := (ssCov'_r)_\lambda$  for  $1 \leq \lambda \leq \Lambda_r$  we get a bijection  $Z_r$  from  $SSPCov_r$  to  $SSCov'_r$ .

In order to guarantee that  $SSPCov_r$  is a covering we have to reduce the universes of the concepts in  $Cset_{pr}$  as follows:

$$UC_{new} = UC \cap (\bigcup \{(ssPCov_r)_\lambda \mid 1 \leq \lambda \leq \Lambda_r\})$$

for all  $C \in Cset_{pr}$ . Then  $SSPCov_r$  will be a subset covering of the discrete union of the new universes. If we want the universes of the new concepts to be surjective on the domains, which means that the corresponding maps are defined on the whole domains, then the domains should be modified as follows:  $DC_{new} = p_1(UC_{new})$  for all  $C \in Cset_{pr}$ . Defining  $Cset_r := \{C_{new} \mid C \in Cset_{pr}\}$ , where  $C_{new}$  has the universe  $UC_{new}$  and the domain  $DC_{new}$ , the bijection  $Z_r$  from  $SSPCov_r$  to  $SSCov'_r$  represents a concept transformation from  $Cset_r$  to  $Cset'_r$ . Truth- and false-sets for the  $C_{new}$  are obtained by  $TS(C_{new}) = DC_{new} \cap TS(C)$  and  $FS(C_{new}) = DC_{new} \cap FS(C)$ .

Two concluding remarks are in place as to the kind of Mathematics here encountered and as to the kind of modeling SPL precedents.

- The mathematical structure presented by this paper is based on only elementary Set Theory that in principle is extremely simple. But any mathematician familiar with the foundation of Set Theory knows how cumbersome it was to clarify its principally simplest part, its axiomatization - which took dozens of years. Not to speak of the axiomatization of Geometry, as this took two thousand years.
- A similar kind of mathematical principal simplicity, initially being difficult to detect and grasp, is encountered in the here preceding presentation of the mathematical structure for an invention over a set of inventive concepts generating it, i.e. in scientizing SPL precedents, i.e. in eventually axiomatizing it. But today, axiomatizing a new plainly mathematical theory using set theory only, i.e. being fully "sub-physics" as it is the case in SPL precedents, is a routine activity of a pertinent mathematician. Hence, it is not a mathematical difficulty to achieve the scientification/axiomatization of SPL precedents [6] but to detect the amenability of Highest Courts' SPL precedents to mathematical scrutiny alias mathematical describability/representability/ modelability - as normally striving for consistency and predictability - and how to leverage on this amenability. As indicated by the above quoted reference, a follow-up paper will elaborate on the axiomatization of inventivity/creativity as implied by SPL [74] - to be distinguished from inventivity/creativity as implied by copyright and by trade mark laws [35].

### 3 An Invention's Mathematical Structure

Chapter 2 has presented the mathematical definition of the mathematical structure over an invention's set of all its inventive concepts. Thereby other aspects of this mathematical structure have intentionally been left vastly away for focusing on assuring its mathematical well-definedness, in spite of the evident implication that this complex new structure then would be questioned as to its usefulness by any mathematician as well as by any patent professional (if the latter were interested in a scientification of this area, at all, as otherwise it would ignore this paper anyway). Nobody from these both target groups of this presentation of this mathematical structure namely has had the faintest chance to grasp thereof its really convincing usefulness.

Before Chapter 4 can show this usefulness, this Chapter 3 must explain the steps to be performed for establishing for a given invention its specific mathematical structure defined by Chapter 2. Establishing it is indispensable for setting up an invention's test under SPL, as also briefly outlined in Chapter 4. This explanation starts with two important disclaimers. The very first two steps in establishing this specific mathematical structure are assumed to be performed by the user of the patent technology presented here prior to this use. These iteratively to be executed - if necessary repeatedly, also after having started using this patent technology - first steps, tightly interwoven with each other, just as with the here presented patent technology, are:

- Marking-up all items of information, factually or legally relevant for this mathematical structure, in all documents technically disclosing the invention (i.e. the patent specification just as documents specifying the pertinent skill, normally pertinent textbooks) and/or describing the SPL and/or Highest Courts' SPL precedents and/or a nation's other laws to be applied in establishing this mathematical structure - i.e. all items of information, factually or legally relevant for defining all the inventive concepts of the invention at issue.
- Identifying all these inventive concepts of the invention at issue, as disclosed by these "marked-up units of information, MUIs" in these documents. In this case they are explicitly disclosed, for the posc (see above), in the documents used for this analysis. Alternatively, an inventive concept may be recognized by the posc also from the invention's specification without using the mark-up therein. Thereby the process performed by the posc of inferring the semantics of the technical teaching from this raw specification, is not discussed here. But this inference process may require from the posc investing into it some reasonable amount of effort, i.e. as usual in reading skillful subject matter presentations not exceeding the level of skill about the subject matter of the invention at issue, thus need not be trivially recognizable.

As of the above SPL-Initiative of the Supreme Court, the result of the test of an invention for its satisfying SPL will be determined by such inventive concepts, themselves being subject of this test, as outlined in Chapter 4. Classical patent jurisprudence has vastly underestimated the complexity of this SPL test and hence the scrutiny indispensable for performing it in a legally and fac-

tually/logically unquestionable and complete manner. Otherwise consistency and predictability in SPL precedents as to emerging technologies invention is not achievable, as shown by the current situation in many SPL court cases. By Advanced IT in the FSTP project achieved insights into the SPL problem, i.e. by the mathematical structure presented in Chapter 2, yielding in the FSTP-Test [36,71], this confusion ought to be overcome.

This mathematical structure assumes that all inventive concepts of the invention, as disclosed by its specification, are identified and used in claiming protection by SPL, although a subset of them may in some cases suffice for defining the scope of this claim, i.e. the scope of SPL protection. Otherwise it is impossible to resolve seemingly insoluble ambiguities - inevitable but up-front not recognizably coming up with some inventions - as outlined below.

To summarize these disclaimers: How to perform these two steps is outside of the scope of this paper. They may be substantially supported by tools developed for this purpose but belonging into an area of Semantics research not being discussed here, yet. The main reason being that such tools would to some degree be depending on the subject area of the invention at issue - in particular those tools for the second one - while the patent technology presented here is subject area independent. Thus, as to performing these initial nontrivial two steps, the user is completely left on his own, today - though it normally would get substantial feedback as to both steps by incrementally testing the invention under SPL (outlined in Chapter 4) and thus discover actual and/or potential deficiencies as to both of them in terms of SPL.

As mathematically introduced by Chapter 2, three levels of abstraction for the representations of all inventive concepts of an invention are defined, as usual in Semantics [25]. Next, the representation of these semantics and their preciseness associated with these three levels is explained in some more detail. In Chapter 4 then also is indicated, how subsets of an invention's concept set must be limited for being meaningful as well as how to define different interpretations of a claimed invention, defining also the scope of this claim claiming it <sup>1</sup>.

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<sup>1</sup>It is important to see already here - this has been presented in [58,73] and will be mathematically elaborated on in detail in [74] - that a patent may comprise several independent inventions, each of them claimed by an independent claim of the patent. In particular in patents dealing with emerging technology inventions - being just model based, as explained above - such a claim(ed invention) may have several different interpretations, each of them identified by its BID-inC generating it. These different interpretations of this claim(ed invention) are called isomorphic iff their respective generative BID-inCs are isomorphic. For any claim(ed invention)'s interpretation exactly one scope is defined, thus also identified by its generative BID-inC.

This raises for emerging technology inventions several important new questions - actually, these question existed for classical technologies inventions, too, but due to their tangibility/visibility never became virulent or at least noticed - such as: How to prevent granting preemptive claim(ed invention)s and/or how to separate patent-eligible from non-patent-eligible issues in a claim(ed invention) and/or what makes different but isomorphic interpretations of a claim(ed invention) patent-eligible and patentable separately from each other (i.e. treat them as non-isomorphic), ...?

But, let's put it simple. First, the below 3 bullet points colloquially describe the 3 levels of granularity of the notional resolution of their inventive concepts alias "inCs" of the invention at issue. Thereafter, paragraphs 10-12 describe the 3 levels, once more and more precisely, in terms of the mathematical structure introduced in Chapter 2. For any level trivially holds that a concept is inventive iff skill does not know a set of concepts and how to combine them such that this combination factually is equivalent to the former concept.

- *OCset* is defined to denote and comprise the invention's inventive concepts, the O-inCs, disclosed for the posc by reading the original documents containing them and grasping the technical teaching they infer within it. I.e.: The layman does not exist in this context - its understanding of the invention or of the specification or of the claim or of the terms therein is completely irrelevant, though it may coincide with that of the posc. As explained elsewhere, e.g. [6,7,71] and outlined in Chapter 4, the meaning of O-inCs in isolation is often principally not definable precisely, as it may depend on the invention's set of concepts selected on the BED level, as indicated in Chapter 2 by paragraph 3 - but even for a claim(ed invention) with only a single interpretation the meanings of compound O-inCs are, due to natural language deficiencies, often blurring/imprecise/indefinite.

- *BADset* is defined to denote and comprise the invention's binary-aggregated-disclosed inventive concepts, the BAD-inCs. Due to the limitations imposed on their general expressiveness explained in paragraph 3, and in any SPL test case to be approved by the posc, the refined meanings of BAD-inCs (logically modeling their resp. O-inCs) are precise/definite - at least, if their interpretation dependency just mentioned may be disregarded, otherwise only the next step of refining them (then e.g. interpretation dependent or otherwise dependent on other BAD-inCs, as outlined in Chapter 4 and FIG 1) will achieve their preciseness/definiteness, as required anyway for the claim(ed invention)'s SPL test.

- *BEDset* is defined to denote and comprise the invention's binary-elementary-disclosed inventive concepts, the BED-inCs. As to their refinement explained in paragraph 3 holds that the refined meanings of BED-inCs (conjunctions of which logically model the BAD-inCs) are precise/definite - possibly achieved by defining for components of BAD-inCs different BED-inCs, e.g. being interpretation specific (see footnote 1). But the fundamental requirement to be met by the claim(ed invention)'s inventive concepts on this level of notional refinements is that they are identified/defined such that they show the maximal number of distinctions as to concepts known by skill (i.e. a further refining of a BED-inC into a set of BEDBED-inCs is either factually not possible, at all, or all these BEDBED-inCs were known by skill already just as how to combine them such as to be factually equivalent to BED-inC, which contradicts BED-inC being inventive by the definition of in-Cs). Next is shown, by paragraphs 10-12, that the mathematical structure defined in Chapter 2 assures that the total information represented on the 3 levels always is the same and that the respective various chunks of this information on the 3 levels are properly mapped onto each other by concept transformations, as described in paragraphs 3-9.

10. *OCset*: The concepts in *OCset* are based on the *mark-up items*, *MUIs*, taken in accordance with the legal requirements from the patent application *doc<sub>0</sub>*. The legal requirements provide the rules from which parts of *doc<sub>0</sub>* the *MUIs* can (must) be taken: Denoting by *SMUI* the set of all *MUIs* selected from *doc<sub>0</sub>*, sets *SSMUI<sub>h</sub>*,  $h = 1, \dots, H$ , of subsets of *SMUI* are identified as clusters for the domains *DOC<sub>h</sub>* of the O-level concepts *OC<sub>h</sub>*,  $h = 1, \dots, H$ . If in a cluster more than one copy of the same set of *MUIs* will be of relevance, because this set will have different interpretations in *BADset*, then we have to distinguish these copies by adding labels. Having selected all *SSMUI<sub>h</sub>* there may be a non-empty remainder

$$RM := SMUI \setminus \bigcup \{ \bigcup \{ sMUI \mid sMUI \in SSMUI_h \} \mid h = 1, \dots, H \}.$$

Then *OC<sub>h</sub>* is defined as follows: *TS(OC<sub>h</sub>)* is given by the finite set of subsets in *SSMUI<sub>h</sub>* having a meaning with respect to the corresponding concept *BADC<sub>h</sub>* and *FS(OC<sub>h</sub>)* :=  $\{RM\} \cup (SSMUI_h \setminus TS(OC_h))$ . The rules for selecting the sets in *TS(OC<sub>h</sub>)* will be made more precise in connection with the related concept *BADC<sub>h</sub>* in *BADset*. Having defined the partition for *D(OC<sub>h</sub>)* into a truth- and a false-set, all other data for *OC<sub>h</sub>* can be concluded easily as has been explained in paragraph 2.

11. *BADset* and *OCset* to *BADset*: The concepts in *BADset* are set up in bijective correspondence  $M_{OAD} : OCset \rightarrow BADset$  with those in *OCset*. *BADC<sub>h</sub>* =  $M_{OAD}(OC_h)$  is the conceptual reference set of a (possibly) aggregated statement making the corresponding O-level concepts more precise. The elements of the truth-set *TS(BADC<sub>h</sub>)* of the domain of *BADC<sub>h</sub>* are given by sets of *MUIs* belonging to *SSMUI<sub>h</sub>*, each of them being combined with a meaning or technical notion they are related with. These sets of *MUIs* are in bijective correspondence  $M_{OADD_h} : TS(OC_h) \rightarrow TS(BADC_h)$  with the sets of *MUIs* in *TS(OC<sub>h</sub>)*, or more precisely, each  $d \in TS(OC_h)$  is exactly the set of *MUIs* defining  $M_{OADD_h}(d) \in TS(BADC_h)$ . This closes the gap left in the setup of *OCset*, because the selection of the subsets of *SSMUI<sub>h</sub>* for *TS(OC<sub>h</sub>)* is delegated to the selection of sets of *MUIs* for *TS(BADC<sub>h</sub>)* related with some meaning or technical notion. For completing the partition of *D(BADC<sub>h</sub>)* into a truth- and a false-set, we set *FS(BADC<sub>h</sub>)* :=  $\{RM\} \cup (SSMUI_h \setminus TS(OC_h))$ .

All other data for *BADC<sub>h</sub>* can be concluded easily as we have explained in paragraph 2. *OCset* and *BADset* are connected by the following bijections:

$$M_{OAD} : OCset \rightarrow BADset \quad \text{and} \quad M_{OADD_h} : D(OC_h) \rightarrow D(BADC_h), \quad h = 1, \dots, H,$$

where  $M_{OADD_h}$  is extended to the false-sets by the corresponding identity map  $M_{OADD_h}(d) := d$  for all  $d \in \{RM\} \cup (SSMUI_h \setminus TS(OC_h))$ . Hence  $M_{OADD_h}$  is truth-preserving for all  $h = 1, \dots, H$ .

12. *BEDset* and *BADset* to *BEDset*: The concepts (more precisely concept representations) in *BEDset* (binary elementary disclosed) are obtained by disaggregating those in *BADset* into elementary concepts, which could not be disaggregated further in a reasonable way, still represent the properties of the invention and are formulated in a non-ambiguous or definite way.

This procedure is represented as a concept transformation  $Z_{BAD}$  between *BADset*

and  $BEDset$  by a bijection  $Z_{BAD} : SSCBAD \rightarrow SSCBED$  from a subset covering  $SSCBAD$  of  $BADset$  to a special subset covering  $SSCBED$  of  $BEDset$ .  $SSCBED$  is simple in the sense that it is given by the set of universes of the concepts in  $BEDset$ , i.e. each  $ssCBED$  in  $SSCBED$  is the universe of a concept in  $BEDset$ , there are no duplications and all universes of concepts in  $BEDset$  are covered. This implies that the length of the sequence  $SSCBAD$  is given by the number of concepts in  $BEDset$ . Furthermore  $Ind(Z_{BAD}^{-1})$  has to be a map on the concept level. Hence according to the remarks in paragraph 6 the disaggregation of the concepts in  $BADset$  is based on a concept wise subdivision of the union of the universes of the concepts in  $BADset$ . Hence we have a mapping  $Ref : \{1, \dots, \Lambda\} \rightarrow BADset$  such that  $ssCBAD_\lambda \subseteq Ref(\lambda)$ . This leads to a mapping from  $BEDset$  to  $BADset$  given by  $Ref \circ Z_{BAD}^{-1}$ . The covering condition for  $SSCBAD$  implies the covering condition  $UC = \bigcup \{ssCBAD_\lambda \mid Ref(\lambda) = C\}$  for all  $C \in BADset$ .

**Remark.** The case that  $Ind(Z_{BAD}^{-1})$  is a map may be assumed without loss of generality. If by some reason the transition from  $BADset$  to  $BEDset$  involves covering sets  $ssCBAD$ , which have non-empty intersections with universes of different concepts in  $BADset$ , i.e.  $Ind(Z_{BAD}^{-1})$  is not a map, then  $BADset$  and consequently  $OCset$  may be reorganized in a way, such that for the new transformation  $Z_{BAD, reorg}$  the relation  $Ind(Z_{BAD, reorg}^{-1})$  will be a map. The details are straightforward, but they need time and space. Hence they have to be omitted here.

**Remark.** Starting in the way described above  $Z_{BAD}$  does not comprise any information whether or how the  $ssCBAD$  are mapped to the  $U(BEDC)$ . If in each case there is an underlying mapping, which may be a bijection in addition, the condition of being truth-preserving can be imposed on  $Z_{BAD}$ , like it has been explained in paragraph 7.

**Remark:** The condition for a concept to be elementary needs a confirmation. The same applies to the condition that the set of concepts still describes the invention. Though some automatized (and still to be developed) support from semantics may apply, this decision more or less depends on the person of posc.

13.  $BIDset$ , and  $BEDset$  to  $BIDset$ :  $BEDset$  as constructed in paragraph 12 may contain almost similar concepts or concepts having parts, which represent parts of concepts in the same collection. In the next chapter this will be formulated more precisely by the notion of a dependent set of concepts. If the creative parts of  $BIDset$  should pass the independency test being part of the 10 tests representing SPL in the final chapter, concepts leading to dependencies in  $BEDset$  have to be removed from  $BEDset$ . In addition, in order that  $BIDset$  will be able to pass the novelty/nonobviousness test, the creative parts of  $BIDset$  should not include concepts, which are anticipated by all  $doc.i$  representing the prior art under consideration. Hence the creative parts of  $BIDset$ , called  $BIDset$  for short in the following considerations, are obtained by removing concepts from

*BEDset* until we arrive at a set of binary independent, not totally anticipated disclosed concepts. This procedure will be explained in the next chapter. For the following considerations the starting point is just that *BIDset* is a subset of *BEDset*. Applying the restriction procedure introduced in paragraph 9 with respect to *BIDset* to  $Z_{BAD}$  and then to a still to be defined concept (representation) transformation  $Z_{OC}$  from the *O*-representation level to the *BAD*-representation level, we will be able to establish similar transformations for the new concept sets, as we initially had for *OCset*, *BADset* and *BEDset*. This will be described explicitly in the next paragraphs.

14. Adjusting *BADset* to *BIDset*: As a next step we are using the restriction construction given in paragraph 9 for reducing *BADset* to  $BADset_r$ , such that the restriction  $Z_{BAD,r}$  of the concept transformation  $Z_{BAD}$  is a concept transformation between  $BADset_r$  and *BIDset*. According to paragraph 12  $Z_{BAD}^{-1}$  induces a map  $Ind(Z_{BAD}^{-1})$  from *BEDset* to *BADset*. Hence the concepts of the restriction  $BADset_r$  resulting from the restriction of *BEDset* to *BIDset* is given by modifications of the concepts in  $Ind(Z_{BAD}^{-1})(BIDset)$  as follows: The universes of the concepts in  $BADset_r$  are obtained from universes of the concepts in  $(Ind(Z_{BAD}^{-1}))(BIDset)$  by removing all covering subsets mapped by  $Z_{BAD}$  to the universes of concepts in  $BEDset \setminus BIDset$ .  $Z_{BAD,r}$  is just the restriction of  $Z_{BAD}$  to the remaining covering subsets. By definition of the induced concept relation none of the universes obtained in this way can be empty.

More explicitly: For  $C \in BADset$  the modified concept  $C_{new}$  has the domain  $DC_{new} = \bigcup \{DC_{ssCBAD} \mid ssCBAD \subseteq UC, Z_{BAD}(ssCBAD) = UC' \text{ for some } C' \in BIDset\}$ ,

the universe is given by

$UC_{new} = \bigcup \{ssCBAD \mid ssCBAD \subseteq UC, Z_{BAD}(ssCBAD) = UC' \text{ for some } C' \in BIDset\}$ .

$SSCBAD_r$  consists of those  $ssCBAD \in SSCBAD$ , which are mapped by  $Z_{BAD}$  to universes of concepts in *BIDset*, and provides a (concept wise) covering of the universes of the concepts in  $BADset_r$ . Then  $Z_{BAD,r} := Z_{BAD}|_{SSCBAD_r}$  is a bijection, providing a concept transformation between  $BADset_r$  and *BIDset*. If  $Z_{BAD}$  is truth-preserving in the sense of paragraph 7, then  $Z_{BAD,r}$  also is truth-preserving.

15. Adjusting *OCset* to  $BADset_r$ : This is quite obvious. We only have to restrict the inverse maps of the bijections

$M_{OAD} : OCset \longrightarrow BADset$  and  $M_{OAD}D_h : D(OC_h) \longrightarrow D(BADC_h)$

to the restricted sets in  $BADset_r$ , which finally leads to  $OCset_r$ . This also can be used to transfer the covering sets of  $BADset_r$  to  $OCset_r$ , leading in an obvious way to a concept transformation  $Z_{OC}$  between  $OCset_r$  and  $BADset_r$ .

More explicitly: For  $C \in OCset$  the modified concept  $C_{new}$  has the domain

$D(OC_{new_h}) = (M_{OAD}D_h)^{-1}(D(BADC_{new_h}))$  for all  $BADC_{new_h} \in BADset_r$ .

If the number of these concepts is  $H' \leq H$ , then we may change the numbering of *BADset* and *OCset* such that  $BADset_r = \{BADC_{new_h} \mid h \leq H'\}$ . Hence

$D(OCnew_h) = (M_{OAD}D_h)^{-1}(D(BADCnew_h))$  for all  $1 \leq h \leq H'$ ,  
 $U(OCnew_h)$  is obtained by restricting the truth-map of  $U(OC_h)$  to  $D(OCnew_h)$ ,  
 $OCset_r = \{OCnew_h \mid 1 \leq h \leq H'\}$ ,  
 $M_{OAD,r} : OCset_r \longrightarrow BADset_r$  given by  $M_{OAD,r}(OCnew_h) = BADCnew_h$ ,  $1 \leq h \leq H'$ , which is a bijection, having the bijections  
 $M_{OAD}D_h|D(OCnew_h) : D(OCnew_h) \longrightarrow D(BADCnew_h)$ ,  $1 \leq h \leq H'$ , as underlying maps between the domains.

16. So far we have ignored the so-called *elements*  $X_{in}$ ,  $0 \leq i \leq I$ ,  $1 \leq n \leq N$ , describing roughly said the general properties of the patent application, derived from  $doc_0$  in terms of aggregated concepts  $X_{0n}$ ,  $1 \leq n \leq N$  combined with mirror FOL predicates  $\underline{X}_{0n}$ ,  $1 \leq n \leq N$ , and the general properties of the prior art, derived from the documents  $doc_i$ ,  $1 \leq i \leq I$ , in terms of aggregated concepts  $X_{in}$ ,  $1 \leq i \leq I$ ,  $1 \leq n \leq N$  combined with mirror FOL predicates  $\underline{X}_{in}$ ,  $1 \leq i \leq I$ ,  $1 \leq n \leq N$ . Every FOL predicate  $\underline{X}_{0n}$  can be represented by the conjunction of mirror predicates of a uniquely determined subset of concepts in  $BADset_r$ , where these subsets of  $BADset_r$  are mutually disjoint and provide a covering of  $BADset_r$ . Hence we have a surjective map  $El_{BAD} : BADset_r \longrightarrow \{1, \dots, N\}$ , separating  $BADset_r$  into  $N$  mutually disjoint subsets  $BADset_{r,n} = El_{BAD}^{-1}(n)$ ,  $1 \leq n \leq N$ . Composing  $El_{BAD}$  with  $M_{OAD,r}$  and  $Ind(Z_{BAD,r}^{-1})$  we get the same kind of decompositions  $OCset_{r,n}$  and  $BIDset_n$ ,  $1 \leq n \leq N$ , for  $OCset_r$  and  $BIDset_r$  respectively. It is easy to see that the maps  $M_{OAD,r}$  and  $Ind(Z_{BAD,r}^{-1})$  and the chain given by the maps  $Z_{OC}$  and  $Z_{BAD,r}$  decompose element wise in accordance with the decompositions of the sets of concepts.

## 4 The Usefulness of This Mathematical Structure

‘‘Claim construction’’ is a key notion of US SPL precedents. Yet, as to emerging technologies inventions, this classical notion of claim construction has proven to be deficient: In a whole series of CAFC decisions these notional deficiencies led to situations, which its Chief Judge recently - in a case remanded to it by the Supreme Court for reconsideration in the light of Mayo - called irreconcilable within the CAFC. A ‘‘refined claim construction’’, as implicitly required by the Supreme Court’s Mayo decision - for mathematically modeling of which the mathematical structure of Chapter 2 has been developed - completes the established/classical notion of claim construction in the sense describable as follows:

*A claimed invention passes its SPL test*  $\iff$  *it passes the FSTP-Test*  
 $\iff$  *the refined claim construction is construable for it*  
 $\iff$  *the ‘‘refined mathematical structure’’ is construable for it.*

The conclusion is: *A claimed invention passes its SPL test*  $\iff$  *the ‘‘refined mathematical structure’’ is construable for it.*



The next paragraphs outline, by what extensions of the mathematical structure of an invention (as defined in Chapter 2 and established for the invention in Chapter 3) it becomes the refined mathematical structure, which will be mathematically presented in [73]. Yet, the above 3 equivalencies indicate already here: If the mathematical structure modeling a claimed invention is a substantial part of its refined mathematical structure, it also models a substantial part of its SPL test - and therefore is already very useful <sup>2</sup>, though it will unfold its full usefulness only as refined mathematical structure.

What the mathematical structure of Chapter 2 of an invention requires for becoming its refined mathematical structure - i.e. for modeling its refined claim construction, i.e. for modeling its SPL test - is outlined by the next two bullet points: It must

- extend its analysis (in Chapters 2 and 3) of solely an invention respectively its technical teaching (“TT.0”) to the analysis of a PTR, being defined to be a “pair of TT.0 over RS”. Thereby a “reference set” RS is a finite set of prior art documents, doc.i’s, the TT.i’s of which allegedly anticipate TT.0 or make it obvious over a combination of them, and
- complete this extended analysis - and hence the correspondingly extended mathematical structure, as defined by the preceding bullet point - by the subtests FSTP test.o,  $2 \leq o \leq 10$ , of the FSTP-Test (see the list given by **FIG 1**), as the extended mathematical structure for this invention performs only FSTP test.1.

The FSTP-Test in **FIG 1**, is here simply quoted from [36] and hence it cannot be understood completely - in [63,73] it will be described in the mathematical style as the above description of the mathematical structure. Yet its principle of working may be figured out already, here, by footnote 2 and the following 3 hints:

- Any one of the finitely many “set of inCs, SoI” identifies a different interpretation of the claim(ed invention) (see footnote 2), for which the execution of all 10 FSTP test.o is attempted to complete,  $2 \leq o \leq 10$ . The {SoI} of the claim(ed invention) is assumed to be determined prior to starting the FSTP-Test. On any prompt by the FSTP-Test, the user must input into it the information it prompted for. If for a specific claim interpretation alias SoI the execution of one of the 10 test.o cannot be completed, because the user cannot provide the input prompted for by test.o, this SoI is abandoned and another SoI is tested. In any case all SoIs are executed, completely or partially only.
- Any input provided by the user may be augmented by its correctness confirmation by the posc.
- The problem  $P.0^{SoI}$  of the NAIO test is an EPC notion, in the US SPL to be

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<sup>2</sup>It is important to see that, for most inventions, the seemingly plausible implication *Its refined mathematical structure models its refined claim construction*  $\Rightarrow$  *Its mathematical structure models its claim construction* is principally wrong; the reason being that the (classical) claim construction principally has no in-Cs, at all. Practically, any limitation of a claim(ed invention) is the cr-C of one of its in-Cs, but it may have more different in-Cs than such limitations [58].

replaced by the total usefulness modeled by the generative {inCs} (see footnote 2) of the claim(ed invention) identified by Sol.

**FIG 1: The FSTP-Test,**

whereby several of the FSTP test.o are solely indicated by their headlines.

**test.1:** The FSTP-Test is executed for all claim interpretations (see footnote 2), with the **posc justified definite disaggregation** of the compound inventive concepts, after the **posc justified these as definite** for the set of interpretations, Sol, selected in (b)/(c), comprising the steps: It

a) prompts the user for the claim(ed invention)'s and prior art's docs with their **"marked-up items, MUIs"**;

b) prompts for all Sol and for any Sol's  $\forall$

$BAD^{Sol} - \underline{Xin} ::= \bigwedge_{1 \leq Sol.in \leq Sol.IN} BAD - crCin^{Sol.in}$  in doc.i-MUI's,  $0 \leq i \leq I$ ,  $1 \leq n \leq N$ ;

c) prompts for the posc's definiteness justification of  $\forall$  compound inCs in Sol, i.e. of  $\forall BAD - crCin^{Sol.in}$ ;

d) prompts to disaggregate  $\forall BAD - crCin^{Sol.in} \forall 0 \leq i \leq I, 0 \leq n \leq N$  into  $\{BED - cr\underline{Cink}^{Sol.in} \mid 1 \leq k^{Sol.in} \leq K^{Sol.IN}\}$  :

$BAD - crCin^{Sol.in} = \bigwedge_{1 \leq k^{Sol.in} \leq K^{Sol.IN}} BED - cr\underline{Cink}^{Sol.in} \wedge BED - cr\underline{Cink}^{Sol.in} \neq BED - cr\underline{Cink}^{Sol.in'} \forall k^{Sol.in} \neq k^{Sol.in'}$ ;

e) prompts for the posc's definiteness justification of its disaggregation in (d);

f) Set  $K^{Sol} ::= \sum_{1 \leq 0n \leq 0N} K^{0n}$ ,  $S^{Sol} ::= \{BED - cr\underline{C0nk}^{Sol.0n} \mid 1 \leq k^{0n} \leq K^{0N}\}$ , with  $K^{Sol} = |\{BED - cr\underline{C0nk}^{Sol.0n} \mid 1 \leq k^{0n} \leq K^{0N}\}|$ ;

**test.2:** Prompts for justifying  $\forall$  BED-crCs in  $S^{Sol}$ : Their **lawful disclosures**;

**test.3:** Prompts for justifying  $\forall$  BED-inCs in  $S^{Sol}$ : Their **definiteness** under §112.6;

**test.4:** Prompts for justifying  $\forall$  BED-inCs in  $S^{Sol}$ : Their **enablement**;

**test.5:** Prompts for justifying  $\forall$  BED-inCs in  $S^{Sol}$ : Their **independence**;

**test.6:** Prompts for justifying  $\forall$  BED-inCs in  $S^{Sol}$ : Their **posc-nonequivalence**:

a) if  $|RS| = 0$  then  $BED^* - inC0k ::=$  "dummy";

b) else performing **c-e**  $\forall 1 \leq i \leq |RS|$ ;

c) It prompts to disaggregate  $\forall BAD - \underline{Xin}$  into  $\bigwedge_{1 \leq kn \leq Kn} BED - in\underline{Cik}^n$  ;

d) It prompts to define  $BED^* - inCik^n ::=$

either  $BED - inC0k^n$  iff  $BED - inCik^n = BED - inC0k^n \wedge$  disclosed  $\wedge$  definite  $\wedge$  enabled,

else "dummy( $ik^n$ )";

e) It prompts for  $JUS^{posc}(BED^* - inCik^n)$ .

**test.7:** Prompts for justifying by NAIO test (see i) below) on  $(S^{Sol}, P.0^{Sol})$ : TT.0 is **not an abstract idea only**;

**test.8:** Prompts for justifying  $\forall$  BED-inCs in  $S^{Sol}$ : TT.0 is **not natural phenomena solely**;

**test.9:** Prompts for justifying  $\forall$  BED-inCs on  $(S^{Sol}, P.0^{Sol})$ : TT.0 is **novel and nonobvious** by NANO test (see ii) below) on the pair  $(S, \text{if } |RS| = 0 \text{ then } \{BED^* - inCok | 1 \leq k \leq K\} \text{ else } \{BED^* - inCik | 1 \leq k \leq K, 1 \leq i \leq |RS|\})$ ;

**test.10:** Prompts for justifying  $\forall$  BED-inCs in  $S^{Sol}$ : TT.0 is **not idempotent** by NANO test (see ii) below) on the pair  $S' \subseteq S$ .

i) The “**Not an Abstract Idea Only, NAIO**” test basically comprises 4 steps [5,7,10,25,18,19], ignoring RS:

- 1) verifying that the specification discloses a problem,  $P.0^{Sol}$ , to be solved by the claim(ed invention) as of  $S^{Sol}$ ;
- 2) verifying, using the inventive concepts of  $S^{Sol}$ , that the claimed invention solves  $P.0^{Sol}$ ;
- 3) verifying that  $P.0^{Sol}$  is not solved by the claim(ed invention), if a BED-inC of  $S^{Sol}$  is removed or relaxed;
- 4) if all verifications 1)-3) apply, then this pair (claim(ed invention),  $S^{Sol}$ ) is “not an abstract idea only”.

ii) The “**Novel And Not Obvious, NANO**” test basically comprises 3 steps, checking all “binary anticipation combinations,  $BAC^{Sol}_s$ ” derivable from the prior art documents in RS for the invention defined by  $S^{Sol}$ :

- 1) generating the  $ANC^{Sol}$  matrix, its lines representing for any prior art document  $i, i = 1, 2, \dots, I$ , the relations between its  $invention^{i.Sol}$ 's BED-inCs to their peers of  $TT.0^{Sol}$ , represented by its columns;
- 2) automatically deriving from the  $ANC^{Sol}$  matrix the set of  $\{AC^{Sol}_s\}$  with the minimal number  $Q^{plcs/Sol}$ ;
- 3) automatically delivering  $(Q^{plcs/Sol}, \{AC^{Sol}_s\})$ , indicating the creativity of the pair (claim(ed invention), Sol).

## 5 References

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